A METHOD FOR THE CALCULATION OF THE GROWTH OF FISHES FROM SCALE MEASUREMENTS
(From the Department of Zoology,
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## Abstract

Huxley's formula for heterogonic growth appears to fit the scale/body length relationship in fish when a correction is made for the growth of the fish previous to the laying down of the scale. Demonstrations of the applicability of the formula are given in the form of plots of scale/body relationships on a double logarithmic grid. A calculator for obtaining fish lengths from scale measurements is described which applies Huxley's formula for heterogonic growth.

## INTRODUCTION

An application of Huxley's formula for heterogonic growth to the scale/body relationship in fishes is introduced here. There is also a description of a calculator that makes use of this formula in the calculation of the size of fishes corresponding to diameters of the annuli of their scales laid down in the years of their existence previous to capture.

The excellent reviews of Graham (1928), Van Oosten (1929), and Lee (1920) render idle any discussion here of the history and applications of the scale method to growth calculations. However, mention should be made of the logarithmic formula introduced by Monastuirsky (1926) which differs from the one proposed here only in that it makes no allowance for the length attained by the fish before the scale is laid down.

Huxley (1932) has shown that the many cases of differential growth which he investigated could be expressed by one general formula which may be written as:-

$$
y=b x^{a}
$$

where $y$ is a dimension of one part
$x$ is a dimension of the other part minus any dimension it might have had at the inception of part " $y$ "
$b$ and $a$ are constants.
The logarithmic form of this equation:-
$\log y=a \log x$ plus $\log b$,
gives a straight line in those cases where the constants " $a$ " and " $b$ " remain the same throughout the life of the organism. This


Figure 1.-The relation between dorso-ventral diameter of the scale and body length in Pomolobus pseudoharengus. Data from Huntsman, 1918. $x=$ 2.5 cm .


Figure 2.-The same data as in figure 1 plotted on a logarithmic scale.
appears to be the nature of the relation existing between growth of the scales and of the body in fishes.


Figure 3.-The logarithmic relationship in various species. For sources of data see text.
Figure 1, taken from Huntsman (1918), fig. 11, which shows the relation between a linear dimension of "side" scales and the standard length of the body in Pomolobus pseudoharengus, may be taken as an illustration of a curve fulfilling the arithmetical form of the equation. From the information afforded by figure 1, figure 2 has been plotted on logarithmic paper, the length of the body at the time of scale formation being the point where the curve in figure 1 cuts the " $x$ " axis. Figure 3 gives further examples of
the relationship in its logarithmic form plotted from data presented in various papers. The data for Clupea harengus, Tautogolabrus adspersus, and Pseudopleuronectes americanus were taken from Huntsman (1918), for Alosa sapidissima from a curve given in Leim (1924), for Oncorhynchus nerka from Dunlop (1924), for Gadus callarius from the measurements by Duff (1929), and for the lake Huron race of Leucichthys artedi from Van Oosten (1929). In most cases surprisingly good approximations to straight lines have resulted, the greatest divergence being at one point in the curve for Clupea harengus at a place where the line had apparently been drawn slightly askew. It should be noted, too, that many of these curves include measurements made on quite small fish so that extrapolation has not played an unduly great part in the derivation of the logarithmic lines.

## Acknowledgments

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## Description of Apparatus

The apparatus is based on the following principles. If two arithmetic series are compared on ordinary graph paper a straight line will result, and the two series may be compared according to the principle of similar triangles on a proportionality machine such as is described in Lea (1915). This is constructed as shown in figure 4. To use this the movable arm is swung on its pivot until its edge coincides with " $x$ ", a chosen point on one series. The other series is placed on a vertical, moving scale and slid along the baseline until the corresponding point " $x_{1}$ " coincides with the edge of the arm. Swinging the arm now cuts off proportional intervals on the two scales.

Similarly two series which give a straight line when plotted

against each other on logarithmic paper may be compared on a proportionality machine if the fixed and moving scales be divided into logarithmic instead of arithmetic units.

Now the case of the arithmetic series discussed in the paragraph above is a particular case involving two series starting out simultaneously from zero. In all other cases the base line has to cut the fixed and moving scales at points which correspond with one set of values fulfilling the ratio between the series under comparison.


This is always so when logarithmic scales are being dealt with, for $\log$. zero is minus infinity. This may perhaps best be made clear by an example. Figure 5 shows the relation of the scales with respect to the body with baseline set to fit the case of Pomolobus pseudoharengus illustrated in figures 1 and 2. In this instance the baseline cuts the "scale diameter" scale at 1.4 mm . and the "body length attained since scale formation" scale at 2.5 cm . This point is indicated on the graphs by an asterisk. Lines " $a^{1 "}$ " " $a^{2}$ ", and " $a^{3}$ " illustrate positions of the hairline pivoted at " $D$ "' which


Figure 6
cut off points on the two scales having values corresponding to those of the solid dots shown on figures 1 and 2.

It is possible, then, by choosing a suitable scale, to compare by proportions two series whose logarithms give a straight line relationship. The evidence submitted in the introduction indicates that fish scale and body growth conform to such a relationship. It remains only, then, to get a magnification of a fish scale in logarithmic units. To do this it is necessary to turn the projected image from its normal arithmetic scale into a logarithmic one by some graphical method. Such a curve is plotted in figure 6.

To avoid having to use a multiplicity of such curves and to be able to use within limits any convenient magnification, a second
proportionality machine is used that reduces images to a definite magnification. In the case of the apparatus described this magnification is 20 diameters. This proportionality machine works in the following manner (see figure 7). " $A$ " is a fixed scale erected at " $O$ " at right angles to the baseline " $O D$ ". " $B$ " is a moving arm pivoted at " $D$ " so that a hairline along " $B$ " swings about " $D$ ". The line " $O D$ " is divided into 20 equal divisions. Now if a

perpendicular be erected at " $E$ " a distance of " $x$ " units from " $D$ ", the distance " $E F$ " above the baseline, that is cut off along it by the hairline, is to " $C O$ ", the distance cut off along " $A$ " by the hairline, as " $x$ " is to 20 . Accordingly if " $E F$ " be " $x$ " times the natural dimension of a length then " CO " will be the natural dimension magnified by 20 diameters and it follows as a corollary that if an image of known magnification be erected at the unit along " $O D$ " corresponding to this magnification it may be magnified to 20 diameters on the scale " $A$ ", and if " $C O$ " be divided into units 20 times a natural dimension the natural size may be read directly on it.

Now if " $A$ " be used as the abscissa of a graph showing the relationship between " $x$ " and log. " $x$ " then the logarithm of " $x$ "
can be read off along "OM" or in effect, "OM" is a logarithmic scale (figure 8 ). Then by pivoting an arm similar to " $B$ " on " $A$ " at " $C_{1}$ " the scale on " $O M$ " could be compared with another logarithmic scale $\left(O_{1} M_{1}\right)$ as in figure 9.

To be able to compare different series, the scale " $M O^{\prime}$ " must be


Figure 9
able to change its position with respect to " $C_{1}$ " and " $O_{1}$ ", and the baseline " $C_{1} O_{1}$ " must be shifted along the scales " $M O$ " and " $M_{1} O_{1}$ ".

These principles have been incorporated in the apparatus in the following manner. The arrangement of the two proportionality machines, and the log. curve is shown in the sketch given with the plan of the apparatus. This sketch is lettered as in figures


7, 8 , and 9 . A study of it will show several modifications of the relations shown in figure 9. The baseline "OC" of the log. curve has been laterally inverted and the point " $O$ " is shackled to the arm " $B$ ". The curve is so mounted that it may pass under the line " $M D$ ". The point where the curve cuts the line " $M D$ " is the logarithm of the distance "OC". The position of the scale " $M O$ ", figure 9 , with respect to the point " $C_{1}$ " has been fixed, and the point where it cuts the baseline has also been fixed at log. 0.1 mm . The first cycle ( 0.1 to 1.0 ) of the curve above the baseline has been omitted and the space used to accommodate the part of the apparatus used for obtaining a standard magnification (see figure 7). The scale " $M_{1} O_{1}$ " on which body length is read has a step in it that leaves a free space between " $C$ " and " $D$ ". The distance between " $C_{1}$ " and " $O_{1}$ " may be varied and the height of " $M_{1}$ " above " $O_{1}$ " may also be varied.

## Operation

The scales are projected on to filing cards after the manner described by Lea (1915). A number of scales of fishes of different sizes, taken from a particular region of the body, are measured, the measurement taken being dependent on the species. For cycloid scales a diameter would be used, in the case of fishes with ctenoid scales, the anterior radius would be measured. These data are used to plot a curve between scale size and body length as was plotted in figure 1. From this graph the average size of the fish at time of scale formation can be obtained by extrapolation. The same material is then plotted on logarithmic paper using this time "body length minus length at time of scale formation." This should give a straight line as in figure 2. It may be necessary to add to or subtract from the first extrapolated value for a size of fish at scale formation to get a straight line. This straight line is then extrapolated so that the amount the fish has grown since the formation of the scale while the scale has been attaining the length of 0.1 mm . may be read.

The scale " $M_{1} O_{1}$ " is moved perpendicular to " $C_{1} O_{1}$ " until the point read at " $O_{1}$ " is the size of the fish attained since scale formation corresponding to a scale length of 0.1 mm . " $M_{1} O_{1}$ " is then


Figure 11.-Photograph of apparatus.
locked in this position. A card with scale measurements projected along its edge is erected perpendicular to " $C D$ " with the edge on which the image has been projected at the unit on " $C D$ " corresponding to its magnification. The lower boundary of the image is placed on the line " $C D$ ". " $B$ " is swung down until the hairline cuts the edge of the card at the upper boundary of the image, moving the log. curve with it. The arm " $C_{1} M_{1}$ " is swung down until its hairline lies over the point " $M$ " where the log. curve cuts the line " $M C D$ ", then the point " $O_{1}$ " is approached to or withdrawn from " $C_{1}$ " until " $C$ ", " $M$ ", and the length of the fish minus the average length at scale formation, " $M_{1}$ ", are in line.

To read the fish length then at the end of year " $x$ " the card with the image on it is slipped down until the lower boundary of this annulus is on " $M C D$ ". " $B$ " is swung down until the hairline touches the upper boundary, " $O$ " follows the hairline. The arm " $C_{1} M_{1}$ " is swung down so that the hairline on it cuts " $M C$ " at the same point as the log. curve does in its new position. On " $M_{1} O_{1}$ " may then be read the length of the fish attained since the time of scale formation. The true length of the fish is arrived at by adding to this figure the amount that represents the growth attained before the scale is formed.

The setting of the scale " $M_{1} O_{1}$ " by moving it perpendicular to " $C_{1} O_{1}$ " is made only once for a given species or race. Movement of the point " $O_{1}$ " with respect to " $C_{1}$ " is made once for each scale from which calculations are made.

This apparatus was first used to make an assessment of the growth history of the Lake Nipissing cisco (Fry, 1937, pp. 63 ff .). A particular study was made of the male fish taken at one particular station in three successive years. It was found that (table 18, Fry, 1937) regardless of the age of the fish when captured the calculated values for the length of the fish at the formation of the first annulus, in a given year class, agree within the limits of probable error. Calculations at subsequent annuli showed Lee's phenomenon but there is evidence to indicate that the phenomenon in this instance represents a real difference in the growth rate of individuals making up the sample in different years:

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